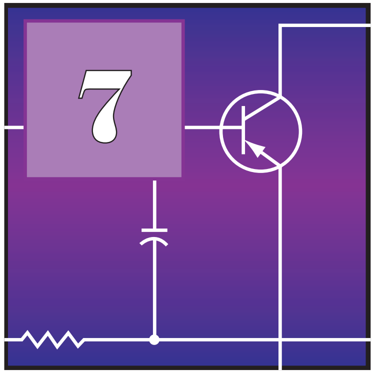


Parallel Circuits

Objectives

After studying this chapter, you will be able to answer these questions:

1. How are electrical components connected in parallel?
2. What are the laws of parallel circuits?

Important Words and Terms

The following words and terms are key concepts in this chapter. Look for them as you read this chapter.

conductance (G)

equivalent resistance

parallel

parallel circuit

siemens (S)

When we were studying the series circuit, the example of the highway between two towns was used. We learned that the several bridges along that highway were in series and each bridge offered a definite amount of resistance to the flow of traffic. As more automobiles are used each day, the need exists to provide better and faster highways. Your state may be constructing new or widened highways so more cars may travel safely without the resistance of narrow bridges and curves. The reason is apparent; more cars can travel on a new two- or three-lane roadway than could travel on the old one-lane road. In fact, twice as many cars can travel on a double road, three times as many on a three-lane road and so on.

If road A has the capacity of ten cars per minute and road B has a capacity of ten cars per minute, then roads A and B together have a capacity of 20 cars per minute, **Figure 7-1.** The bridges on each

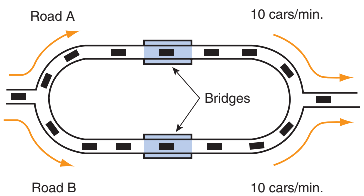


Figure 7-1.
The bridges are in parallel.

road may still offer some resistance to the flow of automobiles.

One could say that these roads are ***parallel*** to each other; that the bridge on road A is parallel to the bridge on road B. This same example can be applied to an electrical circuit. This example will help the beginning student in electricity to understand that in a parallel circuit the resistance decreases because more paths are provided for the flow of electricity.

Equal Resistors in Parallel

In **Figure 7-2**, the highways have been replaced by an electrical circuit. R_1 and R_2 are the bridges (resistance). The automobiles are replaced by electrons flowing along their highways, which are called conductors. At point X, the electrons divide, part taking road A and the other part taking road B. At point Y, the electrons rejoin and continue on their way. Thus, a ***parallel circuit*** has more than one path for current to flow. The parallel combination of resistance units offers less resistance to current than either single resistor.

In **Figure 7-3**, assume that R_1 equals 100 Ω and R_2 also equals 100 Ω. By combining the two in a parallel circuit, the total resistance of the circuit is only 50 Ω. The formula for finding the total resistance of a circuit (R_T) when the resistors are in parallel and all of the same value is:

$$R_T = \frac{R}{N}$$

where R_T is the total resistance, R is the value of any one resistor and N is the number of resistors in parallel.

Apply this formula to **Figure 7-3**,

$$R_T = \frac{100\ \Omega}{2} = 50\ \Omega$$

Like all circuits, there is a voltage, E, across the input terminals. This potential difference (voltage) causes the electrons to flow in the circuit.

Let's draw some conclusions about this circuit.

1. The voltage across all branches or paths of a parallel circuit is the same. In this case, it is the same as the applied voltage (E). You can see that the voltage across R_1 is the same as the voltage across R_2 since the ends of the resistors are connected to the common points X and Y that in turn are connected directly to the power source:

$$E_T = E_{R_1} = E_{R_2}$$

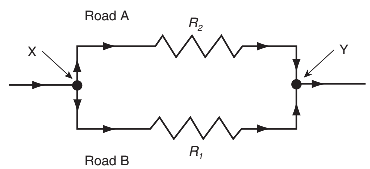


Figure 7-2.
These two resistors are in parallel.

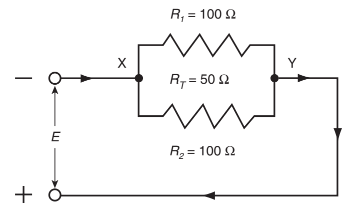


Figure 7-3.
Equal resistances in parallel.

Kirchhoff's voltage law is still true. That is to say, since all components are connected in parallel, there is only one voltage drop. That one voltage drop is equal to the source or applied voltage.

2. The total current in the circuit is equal to the sum of all the currents flowing in the branches of the parallel circuit or:

$$I_T = I_{R_1} + I_{R_2}$$

An example using actual values will help you understand this.

In **Figure 7-4**, total resistance (R_T) of the circuit can be found using:

$$R_T = \frac{R}{N} = \frac{200\ \Omega}{2} = 100\ \Omega$$

The current flowing in the circuit can be found by Ohm's law.

$$I = \frac{E}{R} = \frac{200\ \text{V}}{100\ \Omega} = 2\ \text{A}$$

When the current of 2 amperes reaches point X, it divides. One ampere flows around branch A through R_1, and one ampere flows around branch B through R_2. At point Y, the two currents rejoin.

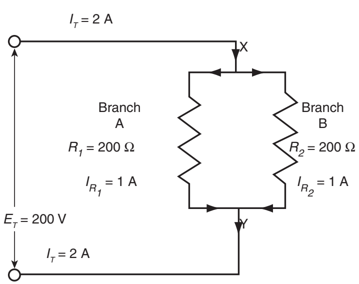


Figure 7-4.
The circuit current divides between the branches of the parallel circuit. Kirchhoff's current law still holds true!

The total current I_T = 2 amps
Current in branch A = 1 amp
Current in branch B = 1 amp
I_T = 1 amp + 1 amp = 2 amps

Again, note that Kirchhoff's other law, Kirchhoff's current law, still holds true. All current that flows into a junction will flow out of it.

Unequal Resistors in Parallel

So far we have assumed that R_1 equals R_2. This is not always so. Often they are unequal, and there is not an equal division of currents flowing in the branches of the parallel circuit.

In **Figure 7-5**, when the current reaches point X, it will still divide, but the greater amount of current will flow through the branch B, because branch B has less resistance than branch A.

Computing the total resistance of a parallel circuit having two unequal resistors is not too difficult. The following formula is called the *product over the sum method* due to the arithmetic operations involved. It can be used for any two resistors in parallel.

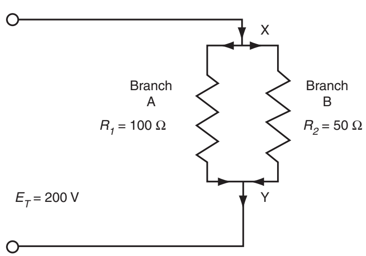


Figure 7-5.
Unequal resistances in parallel.

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2}$$

In our circuit:

$$R_T = \frac{100\ \Omega \times 50\ \Omega}{100\ \Omega + 50\ \Omega} = \frac{5000\ \Omega}{150\ \Omega} = 33.33\ \Omega$$

The total current I_T is then:

$$I_T = \frac{200\ \text{V}}{33.33\ \Omega} = 6\ \text{A}$$

To find how much current will flow in branch A:

$$I_A = \frac{200\ \text{V}}{100\ \Omega} = 2\ \text{A}$$

In branch B:

$$I_B = \frac{200\ \text{V}}{50\ \Omega} = 4\ \text{A}$$

The sum of the branch currents ($I_A + I_B$) is 2 amps + 4 amps = 6 amps. This is the same as the total current flowing in the circuit. This again confirms Kirchhoff's current law.

Conductance

The ability to conduct electricity is opposite the ability to resist the flow of electricity. So we can consider the current carrying ability of any wire or circuit either by stating its resistance to the flow of electrons, or by its ability to conduct electrons. Its ability to conduct is called ***conductance.*** The letter symbol for conductance is ***G.*** Conductance is measured in ***siemens (S).*** Conductance is the *reciprocal* of resistance. That is to say, conductance is one divided by the resistance value. For example, if the resistance of a circuit is 4 Ω, its conductance can be found using:

$$G = \frac{1}{R}$$

so

$$\frac{1}{4\ \Omega} = 0.25\ \text{siemens (S)}$$

History Hit!

Ernst Werner von Siemens (1816–1892)

Siemens is honored with the unit of conductance being named after him. Educated as an electrical engineer in Germany, he led the way for many advances in the principles of electricity. For example, he developed a method to coat wire with an insulation that was seamless—quite an advance for his time. He founded a German company that still carries his name and continues its founder's refinements in the field of electrical devices and generating equipment.

If the circuit resistance is 500 Ω, its conductance is:

$$G = \frac{1}{R} = \frac{1}{500\ \Omega} = 0.002\ \text{S}$$

Two or More Resistors in Parallel

When two or more unequal resistors are connected in parallel, the conductance method of finding R_T is:

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

This formula involves the use of fractions, and is sometimes called the *reciprocal method.* An example will show you how it works. See **Figure 7-6.**

$$R_T = \frac{1}{\frac{1}{100\ \Omega} + \frac{1}{200\ \Omega} + \frac{1}{400\ \Omega}}$$

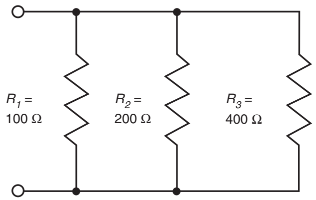


Figure 7-6.
Three unequal resistors in parallel.

Find the least common denominator of the fraction, which is 400.

$$R_T = \frac{1}{\frac{4}{400} + \frac{2}{400} + \frac{1}{400}}$$

$$R_T = \frac{1}{\frac{7}{400}}$$

now,

$$1 \div \frac{7}{400} \text{ is the same as}$$

$$1 \times \frac{400}{7} = \frac{400}{7} = 57.1\ \Omega$$

$$R_T = 57.1\ \Omega$$

Another technique is to simply divide each fraction into one, add the values, then divide into one again.

$$R_T = \frac{1}{\frac{1}{100\ \Omega} + \frac{1}{200\ \Omega} + \frac{1}{400\ \Omega}}$$

$$R_T = \frac{1}{0.01\ \text{S} + 0.005\ \text{S} + 0.0025\ \text{S}}$$

$$R_T = \frac{1}{0.0175\ \text{S}}$$

$$R_T = 57.1\ \Omega$$

Remember to be careful when adding decimal numbers. All decimal points must line up to be added together properly:

$$\begin{array}{r} 0.01 \\ 0.005 \\ +\ \underline{0.0025} \\ \text{Sum} = 0.0175 \end{array}$$

Note: In all problems dealing with resistance in parallel circuits, the total resistance must always be *less* than the value of any resistor in the parallel circuit. Recall that parallel circuits provide additional pathways for current flow. Total resistance always decreases the more parallel resistances are added to the circuit. Use this information to check your work.

Equivalent Resistance

The flow of electricity in a circuit depends upon resistance of the circuit. This resistance can be a single resistor or several resistors connected in series or parallel. Regardless of how many resistors there are or how they are connected, they will combine together to give a total resistance in the circuit. The total resistance is the limiting factor affecting the current. In other words, the total of all resistances might be represented by one resistor value. This resistance is called the ***equivalent resistance*** of the circuit. See **Figure 7-7.**

Step I. Combine R_2 and R_3

$$R_T\ (R_2 \text{ and } R_3) = \frac{400\ \Omega \times 100\ \Omega}{400\ \Omega + 100\ \Omega}$$

$$R_T = \frac{40{,}000\ \Omega}{500\ \Omega} = 80\ \Omega$$

The circuit will appear as shown in **Figure 7-8.**

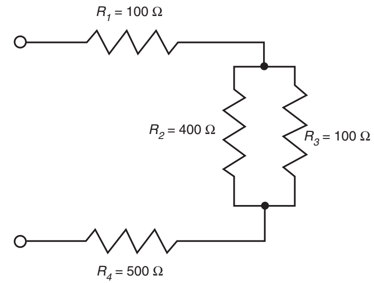


Figure 7-7.
A combination circuit of series and parallel resistors.

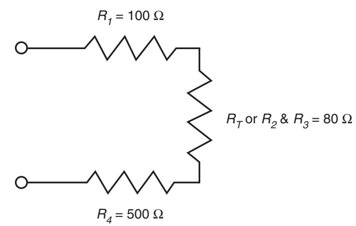


Figure 7-8.
Resistors R_2 and R_3 have been combined.

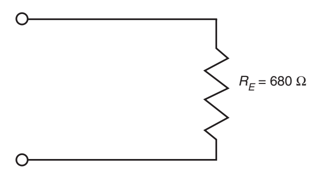


Figure 7-9.
The equivalent resistance of the series and parallel combination circuit.

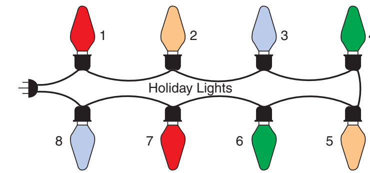


Figure 7-10.
Holiday lights wired in series.

Step II. Combine the three resistors in Figure 7-8. They are in series so:

$$R_1 + (R_2 \text{ and } R_3) + R_4 =$$
$$100\ \Omega + 80\ \Omega + 500\ \Omega = 680\ \Omega$$

The circuit can now be represented by **Figure 7-9.**

Electrically speaking, the circuits of Figures 7-7, 7-8, and 7-9 are exactly the same. R_E (680 Ω) is the equivalent resistance of the combination of R_1, R_2, R_3, and R_4.

Applications

Before leaving the study of series and parallel circuits, let's look at some familiar applications.

A string of holiday or party lights could be connected in a series or parallel manner. Compare **Figures 7-10** and **7-11**.

The symbol  is used for a lightbulb.

In Figure 7-10, the eight lightbulbs are connected in series. All electrical current in the circuit must pass through each lightbulb.

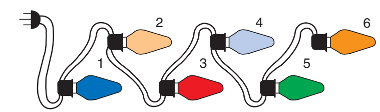


Figure 7-11.
Holiday lights wired in parallel.

Safety Suggestion!

Remember the voltage and current levels present at all electrical connections in your home can be lethal. This means you need to take special safety precautions when exposed to such circuits. The safest action step you can take is to de-energize the circuit you are examining. This is done by removing the fuse or switching off the circuit breaker to the circuit. The electrical voltage (or pressure) coming from a household outlet is sufficient to push enough current through your body to cause death.

If one lightbulb should burn out, all the lights would go out.

Figure 7-12 shows exactly the same circuit as in Figure 7-10 except that conventional schematic symbols are used. This is the way you would find the circuit in the usual electrical drawing.

In Figure 7-11, the same lights are connected in parallel. The current divides between the six branches of the circuit. If one light should burn out, the remaining five will still operate properly.

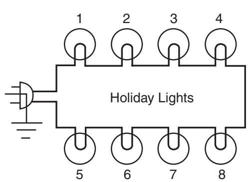


Figure 7-12.
Holiday lights in series using conventional schematic symbols. Note the electrical safety ground symbol found at the plug.

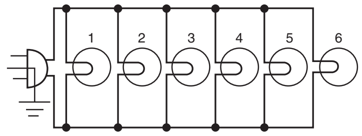


Figure 7-13.
Schematic drawing for holiday lights connected in parallel. Again, note the safety ground symbol found at the plug.

Figure 7-13 shows the schematic drawing for the six holiday lights connected in parallel.

The electrician, in wiring your home, wired all your lights, convenience outlets, and appliances in parallel. We will now apply our knowledge of parallel circuits. See **Figure 7-14.**

Here we have two convenience (duplex) outlets or receptacles wired in parallel across the line voltage. The 110 volt line is made up of two electrical conductors and a safety ground. The conductors are referred to as the hot and neutral wires. The hot wire, usually protected by black (sometimes red) insulation provides the source of the electrical energy. This "hot"

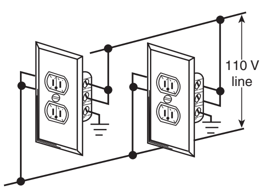


Figure 7-14.
Convenience (duplex) outlets in your home are connected in parallel across the line.

conductor is wired to the brass colored screw terminal of the duplex outlet.

The neutral (sometimes called the cold) wire is usually covered with white insulation and completes the circuit to the device plugged in. The neutral is termed this because it will ultimately be connected to an earth referenced ground. This ground connection can be the cold water pipe (as long as it is metal) or a copper rod pounded into the ground (earth) near the home electrical panel. The neutral wire is connected to the silver colored screw terminal on the duplex outlet. If you look carefully at the front of an outlet, the neutral wire is connected to the larger opening, the hot wire to the smaller opening.

The safety ground wire, or ground, is covered with green insulation. Its purpose is to conduct electrical current away from a fault or short circuit. Let's suppose an electrical heater that is made of metal develops a problem or fault right where the electric cord enters the device. The insulation has been damaged, and the hot wire is touching the metal cabinet. In this unsafe situation, someone touching the metal cabinet of the heater and an electrical ground (another appliance, plumbing, or any other device that may be grounded) could receive a fatal electrical shock. The safety ground takes this potential path of current (from the metal case) to the earth ground. The ground wire provides a low resistance connection from the hot wire to ground. In this way, an excessive amount of current flows. Enough current should flow that it will blow out a fuse or trip a circuit breaker. For this system to function properly, those who do the wiring, electricians, must follow strict codes and rules for everyone's safety. The National Electrical Code specifies the rules that must be followed by electricians who work in homes and industries.

In Figure 7-14, no electric current is being used because no appliance or light (an electrical load) has been plugged in.

When a load is plugged in ($R_1 = 100\ \Omega$), a current flows, **Figure 7-15.**

$$I = \frac{110\text{ V}}{100\ \Omega} = 1.1\text{ A}$$

and the power is:

$$P = I \times E$$

or

$$P = 1.1\text{ A} \times 110\text{ V} = 121\text{ W}$$

R_1 (100 Ω) is the only resistance in the circuit.

When R_2 (also 100 Ω) is plugged in, R_1 and R_2 are in parallel and the total resistance is:

$$R_T = \frac{100\ \Omega}{2} = 50\ \Omega$$

and the total current flowing is:

$$I = \frac{110\text{ V}}{50\ \Omega} = 2.2\text{ A}$$

The power is:

$$P = 2.2\text{ A} \times 110\text{ V} = 242\text{ W}$$

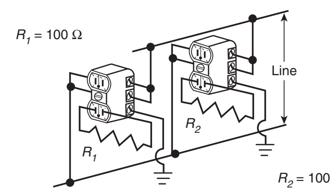


Figure 7-15.
Appliances plugged into convenience outlets are in parallel across the line.

Web Wanderings!

http://www.electronic-circuits-diagrams.com/

The Electronics Zone contains free electronic circuit diagrams with a complete explanation of how the circuit works. Students and hobbyists enjoy reviewing the wide range of circuits listed on this site, as well as reading the free tutorials in basic electronics.

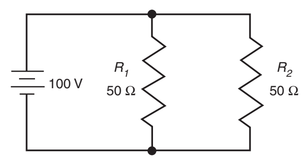
You can now see that as you plug in more and more appliances, lamps, or other electrical loads, the current increases. If current increases, with the voltage at a fixed value (approximately 110 volts), the wattage consumed by these loads is also increased. In this way, your electrical bill also increases. In order to conserve energy and save money, always turn off any unnecessary load, be it the radio, television, or light.

Refer again to the *Project 1—Experimenter* in Chapter 19 of this textbook. In Problem 4, the lights are connected in parallel. Each light burns at the same brilliance as the single light of Problem 2. Because the circuit is a parallel circuit, each light decreases the total resistance of the circuit. The current, of course, will increase (Ohm's law).

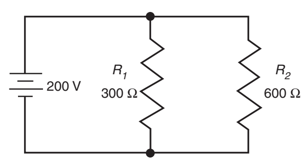
Quiz–Chapter 7

Write your answers to these questions on a separate sheet of paper. Do *not* write in this book.

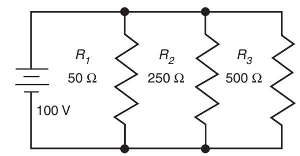
1. Write the formula for total resistance when all resistors in a parallel circuit are equal.
   R_T = ______.
2. Write the formula for total resistance of two unequal resistors in parallel.
   R_T = ______.
3. Write the formula for total resistance of three unequal resistors in parallel.
   R_T = ______.
4. The sum of the branch currents in a parallel circuit is equal to the ______ of the circuit.
5. The voltage across all branches of a parallel circuit is ______.
6. Conductance has the letter symbol ____ and is measured in ____. It is the ____ of resistance.
7. What is the conductance of a circuit that has ten ohms resistance?
8. Find:
   R_T = ______
   I_T = ______
   I_{R_1} = ______
   I_{R_2} = ______



9. Find:
   R_T = ______
   I_T = ______
   I_{R_1} = ______
   I_{R_2} = ______



10. Find:
    R_T = ______
    I_T = ______
    I_{R_1} = ______
    I_{R_2} = ______
    I_{R_3} = ______



11. Find:
    R_T = ______
    E_T = ______